

Automated Inference with Adaptive Batches

Soham De

Joint work with Abhay Yadav, David Jacobs, Tom Goldstein

University of Maryland

OVERVIEW

Most machine learning models use SGD for training BUT...**noisy gradients** & **large variance**

Hard to automate stepsize selection & stopping conditions

OVERVIEW

Most machine learning models use SGD for training BUT...**noisy gradients** & **large variance**

Hard to automate stepsize selection & stopping conditions

In this work: **Big Batch SGD**

Adaptively grow batch size based on amount of noise in the gradients

Easy automated stepsize selection

MOST MODEL FITTING PROBLEMS LOOK LIKE THIS

$$\min \ell(x) := \frac{1}{N} \sum_{i=1}^{N} f(x, z_i)$$

MOST MODEL FITTING PROBLEMS LOOK LIKE THIS

$$\min \ell(x) := \frac{1}{N} \sum_{i=1}^{N} f(x, z_i)$$

We also consider the more general problem:

$$\min \ell(x) := \mathbb{E}_{z \sim p}[f(x, z)]$$

Applications

blah

SVMneural netsIogistic regressionmatrix factorization

SGD



SGD



SGD



SGD

update select data compute gradient $g_t \approx \nabla f(x_t, z_8) \longrightarrow x_{t+1} = x_t - \alpha_t g_t$

SGD



SGD

select data

compute gradient

update

 $g_t \approx \nabla f(x_t, z_8) \longrightarrow x_{t+1} = x_t - \alpha_t g_t$

Error must **decrease** as we approach solution

SGD

select data

compute gradient

update

 $g_t \approx \nabla f(x_t, z_8) \longrightarrow x_{t+1} = x_t - \alpha_t g_t$

Error must **decrease** as we approach solution

classical solution

shrink stepsize

 $\lim_{t \to \infty} \alpha_t = 0$

SGD

select data

compute gradient

update

 $g_t \approx \nabla f(x_t, z_8) \longrightarrow x_{t+1} = x_t - \alpha_t g_t$

Error must **decrease** as we approach solution

classical solution

shrink stepsize $\lim_{t \to \infty} \alpha_t = 0$ hard to pick stepsize schedule





compute gradient

update

 $x_{t+1} = x_t - \alpha_t g_t$

close to optimal solution gets worse









regime I: far from optimal

regime 2: close to optimal

regime I: far from optimal

regime 2: close to optimal

Noisy gradients improve solution

Large batches do unnecessary work + Get stuck in local minima

regime I: far from optimal

regime 2: close to optimal

Noisy gradients improve solution

Large batches do unnecessary work +

Get stuck in local minima

small batches work well!

regime I: far from optimal

Noisy gradients improve solution

ł

Large batches do unnecessary work + Get stuck in local minima

small batches work well!

regime 2: close to optimal

Noisy gradients with high variance worsen solution

Small batches require stepsize decay (hard to tune)

regime I: far from optimal

Noisy gradients improve solution

Large batches do

unnecessary work + Get stuck in local minima

small batches work well!

regime 2: close to optimal

Noisy gradients with high variance worsen solution

Small batches require stepsize decay (hard to tune)

large batches work well!

regime I: far from optimal

regime 2: close to optimal

Noisy gradients improve solution Noisy gradients with high variance worsen solution

Large b unneces Adaptively Grow Batches OverTime!

require rd to tune)

Get stuck in local minima

small batches work well!

large batches work well!

Standard result in stochastic optimization:

Lemma

A sufficient condition for $-\nabla \ell_{\mathcal{B}}(x)$ to be a descent direction is: $\|\nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x)\|^2 < \|\nabla \ell_{\mathcal{B}}(x)\|^2.$

Standard result in stochastic optimization:

LemmaA sufficient condition for $-\nabla \ell_{\mathcal{B}}(x)$ to be a descent direction is: $\|\nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x)\|^2 < \|\nabla \ell_{\mathcal{B}}(x)\|^2$.errorapproximate
gradient

Standard result in stochastic optimization:



If error is small relative to gradient : descent direction

Standard result in stochastic optimization:

LemmaA sufficient condition for $-\nabla \ell_{\mathcal{B}}(x)$ to be a descent direction is: $\|\nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x)\|^2 < \|\nabla \ell_{\mathcal{B}}(x)\|^2$.errorapproximate
gradient

How big is this error?

How large does the batch need to be to guarantee this?

Theorem

Assume f has L_z - Lipschitz dependence on data z. Then, expected error is uniformly bounded by:

$$\mathbb{E}_{\mathcal{B}} \| \nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x) \|^{2} = \operatorname{Tr} \operatorname{Var}_{\mathcal{B}}(\nabla \ell_{\mathcal{B}}(x))$$
$$\leq \frac{1}{|\mathcal{B}|} \cdot 4L_{z}^{2} \operatorname{Tr} \operatorname{Var}_{z}(z)$$

Theorem

Assume f has L_z - Lipschitz dependence on data z. Then, expected error is uniformly bounded by:

$$\mathbb{E}_{\mathcal{B}} \|\nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x)\|^2 = \operatorname{Tr} \operatorname{Var}_{\mathcal{B}}(\nabla \ell_{\mathcal{B}}(x))$$

expected error

batch gradient variance

$$\leq \frac{1}{|\mathcal{B}|} \cdot 4L_z^2 \operatorname{Tr} \operatorname{Var}_z(z)$$

Theorem

Assume f has L_z - Lipschitz dependence on data z. Then, expected error is uniformly bounded by:

$$\mathbb{E}_{\mathcal{B}} \|\nabla \ell_{\mathcal{B}}(x) - \nabla \ell(x)\|^{2} = \operatorname{Tr} \operatorname{Var}_{\mathcal{B}}(\nabla \ell_{\mathcal{B}}(x))$$

expected error batch gradient variance
$$\leq \frac{1}{|\mathcal{B}|} \cdot 4L_{z}^{2} \operatorname{Tr} \operatorname{Var}_{z}(z)$$

batch size data variance

Theorem



Higher Batch Size --> Lower Gradient Variance



From the previous Lemma and Theorem: We expect $-\nabla \ell_{\mathcal{B}}(x)$ to be a descent direction reasonably often provided:

$$\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$$

where $\theta \in (0,1)$

From the previous Lemma and Theorem: We expect $-\nabla \ell_{\mathcal{B}}(x)$ to be a descent direction reasonably often provided:

$$\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$$

where $\theta \in (0, 1)$

We use this observation in Big Batch SGD

BIG BATCH SGD





BIG BATCH SGD

estimate size of error using **variance** of batch gradients

pick batch B


if signal > noise (gradient) (variance) **update**

if signal > noise (gradient) (variance) **update**



if signal > noise (gradient) (variance) **update** pick batch B



if signal > noise (gradient) (variance) **update** pick batch B



if signal > noise (gradient) (variance) **update** pick batch B



if signal > noise (gradient) (variance) **update**



if signal > noise (gradient) (variance) **update**



if signal > noise (gradient) (variance) **update**

otherwise increase batch size



if signal > noise (gradient) (variance) **update**

otherwise increase batch size



if signal > noise (gradient) (variance) **update**

otherwise increase batch size







On each iteration t

• Estimate size of gradient error by computing variance

- Estimate size of gradient error by computing variance
- Pick batch *B* large enough such that $\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$ where $\theta \in (0, 1)$

- Estimate size of gradient error by computing variance
- Pick batch *B* large enough such that $\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$ where $\theta \in (0, 1)$
- Choose stepsize α_t

- Estimate size of gradient error by computing variance
- Pick batch B large enough such that

$$\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$$

where $\theta \in (0, 1)$

- Choose stepsize α_t
- Update

$$x_{t+1} = x_t - \alpha_t \nabla \ell_{\mathcal{B}_t}(x_t)$$

- Estimate size of gradient error by computing variance
- Pick batch B large enough such that

$$\theta^{2} \mathbb{E} \|\nabla \ell_{\mathcal{B}_{t}}(x_{t})\|^{2} \geq \frac{1}{|\mathcal{B}_{t}|} \operatorname{Tr} \operatorname{Var}_{z} \nabla f(x_{t}, z)$$
where $\theta \in (0, 1)$
extra step to SGE

- Choose stepsize $lpha_t$
- Update

$$x_{t+1} = x_t - \alpha_t \nabla \ell_{\mathcal{B}_t}(x_t)$$

On each iteration t

- Estimate size of gradient error by computing variance
- Pick batch B large enough such that

$$\theta^2 \mathbb{E} \| \nabla \ell_{\mathcal{B}_t}(x_t) \|^2 \ge \frac{1}{|\mathcal{B}_t|} \operatorname{Tr} \operatorname{Var}_z \nabla f(x_t, z)$$

where $\theta \in (0, 1)$

- Choose stepsize α_t
- Update

$$x_{t+1} = x_t - \alpha_t \nabla \ell_{\mathcal{B}_t}(x_t)$$

Can be estimated using the batch B

CONVERGENCE

Assumption: ℓ has L-Lipschitz gradients $\ell(x) \leq \ell(y) + \nabla \ell(y)^T (x - y) + \frac{L}{2} ||x - y||^2$ **Assumption:** ℓ satisfies the Polyak-Lojasiewicz (PL) Inequality $||\nabla \ell(x)||^2 \geq 2\mu(\ell(x) - \ell(x^*))$

CONVERGENCE

Assumption: ℓ has L-Lipschitz gradients $\ell(x) \leq \ell(y) + \nabla \ell(y)^T (x - y) + \frac{L}{2} ||x - y||^2$ **Assumption:** ℓ satisfies the Polyak-Lojasiewicz (PL) Inequality $||\nabla \ell(x)||^2 \geq 2\mu(\ell(x) - \ell(x^*))$

Theorem

Big Batch SGD converges linearly: $\mathbb{E}[\ell(x_{t+1}) - \ell(x^*)] \leq \left(1 - \frac{\mu}{\beta L}\right) \cdot \mathbb{E}[\ell(x_t) - \ell(x^*)]$ with optimal step size $\alpha = \frac{1}{\beta L}$ where $\beta = \frac{\theta^2 + (1 - \theta)^2}{(1 - \theta)^2}$

CONVERGENCE RESULTS

Theorem

Big Batch SGD converges linearly: $\mathbb{E}[\ell(x_{t+1}) - \ell(x^*)] \leq \left(1 - \frac{\mu}{\beta L}\right) \cdot \mathbb{E}[\ell(x_t) - \ell(x^*)]$ with optimal step size $\alpha = \frac{1}{\beta L}$ where $\beta = \frac{\theta^2 + (1 - \theta)^2}{(1 - \theta)^2}$

Per-iteration convergence rate

CONVERGENCE RESULTS

Theorem

Big Batch SGD converges linearly: $\mathbb{E}[\ell(x_{t+1}) - \ell(x^*)] \leq \left(1 - \frac{\mu}{\beta L}\right) \cdot \mathbb{E}[\ell(x_t) - \ell(x^*)]$ with optimal step size $\alpha = \frac{1}{\beta L}$ where $\beta = \frac{\theta^2 + (1 - \theta)^2}{(1 - \theta)^2}$

Per-iteration convergence rate

We show:

$$|\mathcal{B}| > \mathcal{O}(1/\epsilon) \longrightarrow \text{Error} < \epsilon$$

optimal convergence in the infinite data case

Controlling noise enables automated stepsize selection

- Controlling noise enables automated stepsize selection
 - Backtracking line search works well!

- Controlling noise enables automated stepsize selection
 - Backtracking line search works well!
 - Stepsize schemes using curvature estimates also work!

- Controlling noise enables automated stepsize selection
 - Backtracking line search works well!
 - Stepsize schemes using curvature estimates also work!
- More accurate gradients enable automatic stopping conditions

- Controlling noise enables automated stepsize selection
 - Backtracking line search works well!
 - Stepsize schemes using curvature estimates also work!
- More accurate gradients enable automatic stopping conditions
- Bigger batches are better in parallel/distributed settings

BACKTRACKING LINE SEARCH

Also referred to as Armijo Line Search

Measures sufficient decrease condition of objective function

For regular (deterministic) gradient descent:

$$\ell(x_{t+1}) \leq \ell(x_t) - c\alpha_t \|\nabla \ell(x_t)\|^2$$

BACKTRACKING LINE SEARCH

Also referred to as Armijo Line Search

Measures sufficient decrease condition of objective function

For regular (deterministic) gradient descent:

$$\ell(x_{t+1}) \leq \ell(x_t) - c\alpha_t \|\nabla \ell(x_t)\|^2$$

new objective

current objective sufficient decrease

BACKTRACKING LINE SEARCH

Also referred to as Armijo Line Search

Measures sufficient decrease condition of objective function

For regular (deterministic) gradient descent:

$$\frac{\ell(x_{t+1})}{\text{new objective}} \leq \frac{\ell(x_t)}{current} - \frac{c\alpha_t \|\nabla \ell(x_t)\|^2}{sufficient \ \text{decrease}}$$

If this fails, decrease stepsize and check again

BACKTRACKING WITH SGD

Measures sufficient decrease condition on individual functions



BACKTRACKING WITH SGD

Measures sufficient decrease condition on individual functions



Moves to the optimum of individual functions, not the global average

BACKTRACKING WITH SGD

Measures sufficient decrease condition on individual functions

Big Batch SGD gets better estimate of the approximate decrease of original objective



Moves to the optimum of individual functions, not the global average

BACKTRACKING WORKS WITH BIG BATCH SGD

Decrease stepsize until:

$$\ell_{\mathcal{B}}(x_{t+1}) \le \ell_{\mathcal{B}}(x_t) - c\alpha_t \|\nabla \ell_{\mathcal{B}}(x_t)\|^2$$

Measures a condition of sufficient decrease using batch B
BACKTRACKING WORKS WITH BIG BATCH SGD

Decrease stepsize until:

$$\ell_{\mathcal{B}}(x_{t+1}) \le \ell_{\mathcal{B}}(x_t) - c\alpha_t \|\nabla \ell_{\mathcal{B}}(x_t)\|^2$$

Measures a condition of sufficient decrease using batch B

Theorem

Big Batch SGD with backtracking line search converges linearly:

$$\mathbb{E}[\ell(x_{t+1}) - \ell(x^*)] \le \left(1 - \frac{\beta \mu}{\beta L}\right) \mathbb{E}[\ell(x_t) - \ell(x^*)]$$

with initial step size set large enough s.t. $\alpha_0 \ge \frac{1}{2\beta L}$

BACKTRACKING WORKS WITH BIG BATCH SGD

Decrease stepsize until:

 $\ell_{\mathcal{B}}(x_{t+1}) \le \ell_{\mathcal{B}}(x_t) - c\alpha_t \|\nabla \ell_{\mathcal{B}}(x_t)\|^2$

We also derive optimal stepsizes for Big Batch SGD using Barzilai-Borwein (BB) curvature estimates, with provable guarantees. Check paper for details.

Big Batch SGD with backtracking line search converges linearly: $\mathbb{E}[\ell(x_{t+1}) - \ell(x^*)] \leq \left(1 - \frac{c\mu}{\beta L}\right) \mathbb{E}[\ell(x_t) - \ell(x^*)]$ with initial step size set large enough s.t. $\alpha_0 \geq \frac{1}{2\beta L}$

Model: Ridge Regression & Logistic Regression



BBS: Big Batch SGD

Model: Ridge Regression & Logistic Regression



BBS: Big Batch SGD

Model: Ridge Regression & Logistic Regression



BBS: Big Batch SGD

Model: Ridge Regression & Logistic Regression



BBS: Big Batch SGD

Model: Ridge Regression & Logistic Regression



BBS: Big Batch SGD

Batch Size Increase



BBS: Big Batch SGD

Stepsizes Used



BBS: Big Batch SGD

4-LAYER CNN

CIFAR-10 (left) & SVHN (right)



TAKFAWAYS

We introduce: Big Batch SGD

Adaptively grows batch size over time to maintain a nearly constant signal-to-noise ratio in the gradients

Better control of the noise makes it easy to automate

Adaptive stepsize methods work well with this method

Better for parallel/distributed settings

THANKS!

Feel free to get in touch!

Extended version on arXiv: "Big Batch SGD: Automated Inference using Adaptive Batch Sizes" <u>https://arxiv.org/abs/1610.05792</u>

email: sohamde@cs.umd.edu website: <u>https://cs.umd.edu/~sohamde/</u>



Soham De



Abhay Yadav



David Jacobs



Tom Goldstein